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SPECIAL BRIEF NOTE

FORCE PREDICTION BY PIV IMAGING: A MOMENTUM-BASED APPROACH

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A momentum-based method for determining the instantaneous force on a body is formulated for application to quantitative imaging of vortex shedding from a cylinder. It involves a control volume that is fixed to the reference frame of an accelerating body and requires only the instantaneous velocity within the control volume and derivatives of velocity along the control surface.

1. INTRODUCTION

The Technique of high-image-density particle-image velocimetry has been successfully employed to provide quantitative interpretation of the instantaneous streamline topology and vorticity fields for vortex formation from stationary and oscillating bluff bodies, as summarized by Rockwell (1993). Such a quantitative approach provides the basis for determining the instantaneous force on the body, as addressed by Lin & Rockwell (1996). This approach, which is based on the concept described by Lighthill (1986), requires only evaluation of the time derivative of the moments of vorticity obtained from PIV data. From a practical standpoint, this technique is useful for oscillatory or wave-type flows where the vorticity remains confined to a finite domain or, in an approximate sense, to flows where diffusion yields rapid decay of vorticity concentrations outside the field of view. The concept described by Lighthill (1986) was extended by Noca *et al.* (1997), in order to account for convection of vorticity away from the region immediately surrounding the body, e.g. mean flow past a stationary cylinder. This was accomplished by starting with a control-volume approach, focusing on a vorticity-based representation, and cleverly eliminating the pressure term. Complete details of this derivation are given by Noca (1996). The predicted lift coefficient for vortex shedding from a cylinder in cross-flow, using the velocity field calculated from a high-resolution Lagrangian technique, showed excellent agreement with conventional approaches.

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The advantage of these vorticity-based approaches is that the relationship between the evolution of the vorticity field and the loading is directly evident. Effective application requires, however, accurate evaluation of vorticity throughout the domain of interest, including the region near the solid boundary of the body. The objective of the present study is to develop an alternative technique that minimizes evaluation of spatial derivatives. It has its genesis in the general framework of Wu (1981), whose theoretical developments for describing loading of single and multiple bodies are linked to control-volume concepts. The present effort complements the parallel, recently completed study of Noca (1997). Taking a different approach, he arrives at a momentum-based control-volume formulation, which is presumably equivalent to the present one. In addition, using a transformation between a volume and a surface integral, he derives an expression for the force requiring temporal and spatial velocity derivatives only on the external surface of the control volume.

2. APPROACHES

2.1. Experimental System and Techniques

A circular cylinder diameter of $D = 50{\cdot}8$ mm was submerged in quiescent water within a large-scale test section. It was subjected to sinusoidal oscillations in the horizontal plane at a value of Keulegan–Carpenter number $\mathrm{KC} = 2\pi A/D = 10$, in which A is the half-amplitude of the oscillation and D is the diameter of the cylinder. The value of the Stokes number is 378, corresponding to a Reynolds number of 3780. The lift was measured by spatially integrating the instantaneous surface pressures at a section of the cylinder immediately adjacent to the laser sheet employed for PIV measurements. The high-image-density particle-image technique described by Rockwell *et al.* (1993) was employed for illumination of the 12 μm scattering particles. Images were acquired using high-resolution 35 mm film at a magnification of $M = 1{:}\,8{\cdot}3$, and at a time spacing $\delta t/T = 0{\cdot}0286$, where T is the period of the cylinder oscillation. These patterns of particle images were then digitized at a resolution of 125 pixels/mm, then interrogated using a single-frame cross-correlation technique to yield the velocity field. The effective grid size of the velocity field was 0·36 mm on the film. Complete details of the experimental system and technique are given by Lin & Rockwell (1997)

2.2. Numerical Technique

The vortex-in-cell method of solving the vorticity transport equation in two dimensions, which is described in detail by Graham (1988) and Meneghini & Bearman (1995), provides a basis for assessment of the control-volume approach described in the following section. Basically, it allows efficient prediction of the instantaneous nature of vortex-dominated flows using a split time-step approach to separate the processes of diffusion and convection. The convection terms in the vorticity transport equation are simulated by convecting point vortices. Velocities at discrete vortex positions are calculated on a polar mesh via solution of Poisson's equation for the stream function. Viscous diffusion is incorporated using a finite difference scheme on the same mesh, which has 128 and 170 divisions, respectively, in the azimuthal and radial directions; a closer mesh spacing is employed near the cylinder in order to resolve accurately the boundary-layer development. For the present test case, the calculated flow corresponds to an oscillating cylinder in quiescent fluid at $\mathrm{KC} = 6$ and a Reynolds number based on the maximum cylinder velocity of 1092. The nondimensional time-step based on the diameter of the cylinder was $\delta t^* = 0{\cdot}0025$; the corresponding time-step normalized by the period T of the cylinder oscillation was $\delta t/T = 0{\cdot}00042$.

2.3. Control-Volume Approach

The lift coefficient on the cylinder was calculated for the numerically based and experimentally (PIV) based velocity fields by considering a control volume fixed to the reference frame of an accelerating body, i.e. cylinder, as indicated in the inset of Figure 1. The control-volume equation reads

$$\mathbf{F} = -\frac{\mathrm{d}}{\mathrm{d}t}\int_{V_F} \rho\mathbf{u}\,\mathrm{d}v - \oint_{S_\pi} [\rho\mathbf{u}(\mathbf{n}\cdot(\mathbf{u}-\mathbf{u}_s)) + p\mathbf{n} - \tau]\,\mathrm{d}s, \tag{1}$$

in which V_F is the fluid volume bounded by the exterior surface S_E of the control volume, and u and u_s are the velocities of the fluid and the control-volume surface. The principal

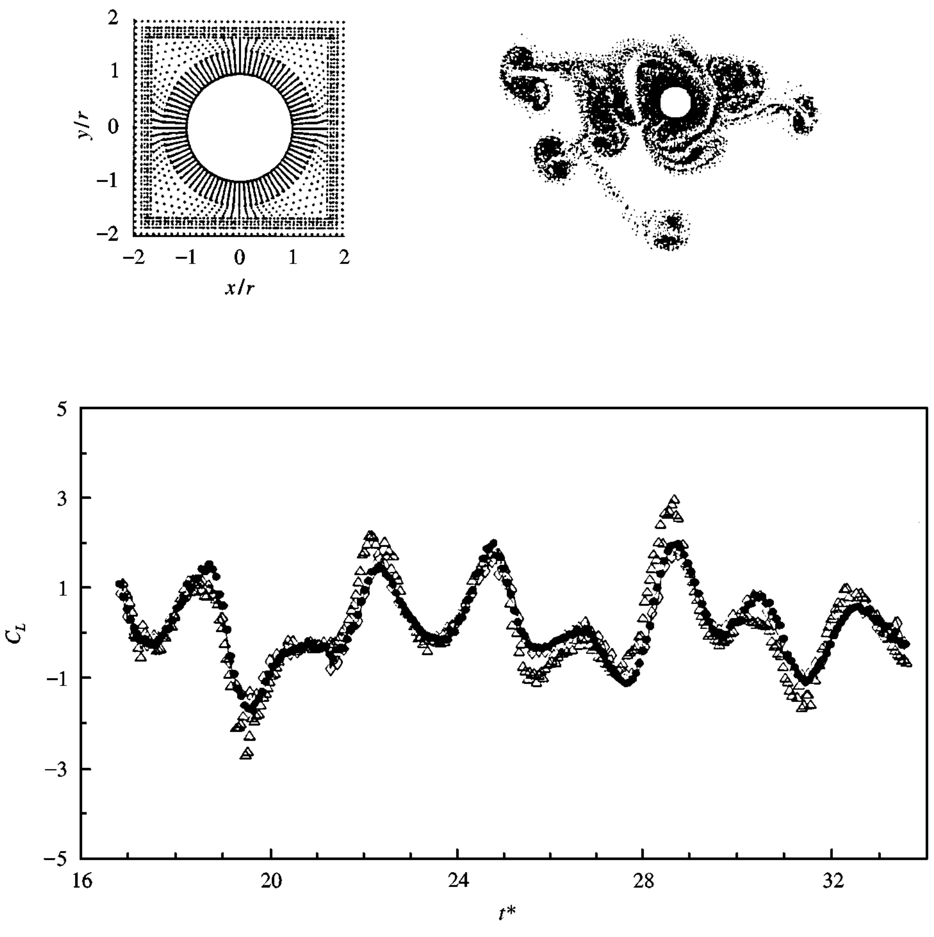

Figure 1. Comparison of lift coefficients determined from flow field calculated by vortex-in-cell technique for KC = 6: ●, Blasius method; △, momentum balance method; ◇–◇, surface stress technique.

contributions to the force are therefore due to (i) the time rate change of momentum within the control volume, (ii) the net momentum flux across the boundaries of the control surface, (iii) the instantaneous pressure force acting on the control surface, and (iv) the instantaneous shear force on the control surface. If the velocity field is known at each instant during the unsteady motion of the cylinder, terms (i) and (ii) can be evaluated. Term (iii) requires the pressure distribution along the control surface. It is obtained by integrating the x and y momentum equations along the x and y surfaces:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right], \tag{2a}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu\left[\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right], \tag{2b}$$

The vector shear stress $\boldsymbol{\tau}$ involves the component $\tau_{yx} = \mu(\partial v/\partial x + \partial u/\partial y)$ along the vertical face of the control volume and $\tau_{xy} = \mu(\partial u/\partial y + \partial v/\partial x)$ along the horizontal face.

The control volume shown in Figure 1 has a composite grid system. The same grid configuration was used for both the PIV and numerical calculations; the spatial resolution for the numerical (CFD) calculations was, however, much finer. The configuration involves (i) a radial region adjacent to the circular boundary of the cylinder; (ii) a rectangular region conforming to the rectangular PIV measurement grid; and (iii) a region that provides a smooth transformation between them. For the PIV-based calculations, the radial region covers a width of 0·225D with 15 and 80 uniform spacings in the radial and azimuthal directions, respectively. The rectangular grid framing the control-volume boundary, which is used for evaluation of momentum flux and pressure at the boundary, involves three squares centred with respect to the cylinder. The grid between the radial and rectangular grids is composed of quadrilateral elements.

2.4. Numerically Based Comparison

Lin *et al.* (1996) provide a comprehensive description of the principal techniques for calculating force coefficients using vortex-based approaches. The methods employed here are: (a) the Blasius technique (Wu 1981) in which the lift involves the time rate of change of the total first moment of the vorticity field in the cylinder and in the fluid; (b) the surface stress technique, involving evaluation of lift due to the pressure and skin-friction contributions at the cylinder surface, calculated from distributions of the vorticity gradient and vorticity along the surface. Comparison of these two well-known techniques with the present control volume approach is shown in Figure 1 over a range of times for which a large number of vortices have been formed; the illustrated vortex pattern corresponds to $t^* = 30{\cdot}2$. Rather than employing a fine grid allowed by the high-resolution numerical data described in the foregoing, the framing-grid resolution for the illustration of Figure 1 was 0·0298D, comparable to that attainable with PIV measurements. The intent is to justify not only the present momentum-based control volume method, but also the use of a coarse grid characteristic of the experimental imaging.

2.5. Experimentally Based Comparison

PIV images of the instantaneous velocity field were acquired for 37 successive instants over the oscillation cycle of the cylinder. Four representative velocity images, the signature of

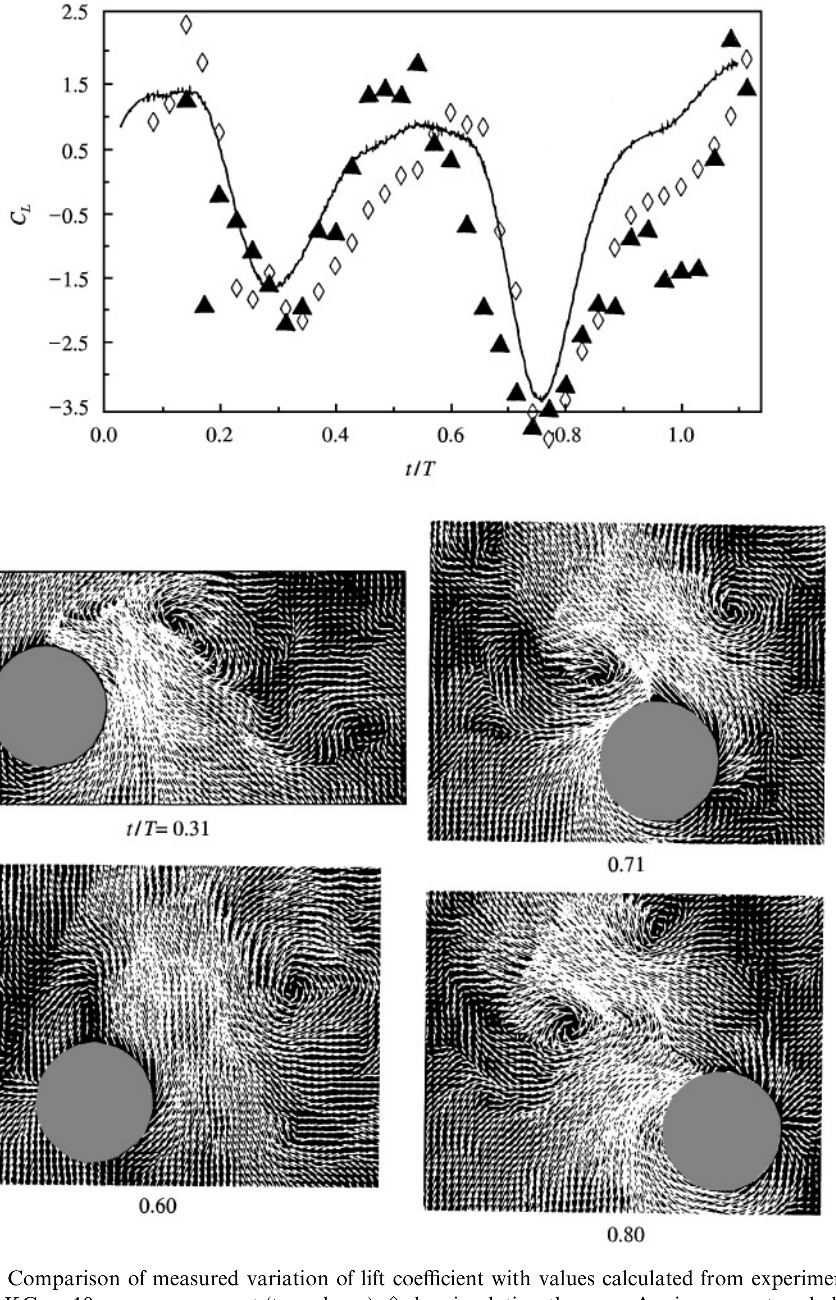

Figure 2. Comparison of measured variation of lift coefficient with values calculated from experimental PIV data for KC = 10: —, measurement (transducer); ◇, by circulation theorem; ▲, via momentum balance.

C_L measured by the pressure transducer system, and the calculated values of C_L determined from the control volume approach are shown in Figure 2. After each of the time histories of the four terms in equation (1) was calculated, a tenth-order polynomial fit was applied to smooth each distribution. The resultant values of C_L are indicated by the black triangles in

Figure 2. The overall trend of the calculated values of C_L, as well as the maximum negative and positive values, are generally approximated. The significant deviations near $t/T = 0{\cdot}15$ and 1·0 are most likely due to three-dimensional effects, whose magnitude is expected to vary during the oscillation cycle of the cylinder. Aside from possible three-dimensional effects, the scatter in the data is due primarily to undersampling, i.e. acquisition of images at a rate of $\delta t/T = 0{\cdot}0286$; the relatively coarse (0·0596D) framing grid is also a source of aberration. These temporal and spatial sources of deviation can be interpreted with reference to equations (1) and (2). Lack of sufficient temporal resolution most significantly influences accurate evaluation of the time derivative of the volume integral, and the coarse spatial resolution substantially influences the evaluation of spatial derivatives along the surface of the control volume. These issues will no doubt be addressed with future developments in PIV-imaging techniques.

Of course, this control-volume approach can also be employed to determine the steady forces on a body by averaging the time sequence of images represented in Figure 2 to obtain a single time-averaged image and employing the simplified form of equation (1), i.e. neglecting the first term on the right-hand side. This type of effort is currently underway.

Alternatively, the instantaneous lift force L was evaluated using the circulation theorem $L = \rho U \Gamma$. It is strictly appropriate only for steady flow. Its remarkable success for ensemble-averaged data in a flow similar to the present one is described by Obasaju et al. (1988). In the present case, the instantaneous value of Γ was found by performing a line integral about the circle defined by $r = 1{\cdot}2R$, where R is the radius of the cylinder. As shown by the diamond symbols in Figure 2, the calculated values exhibit a reasonable approximation.

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